Learn Quantum Mechanics with Haskell

Scott N. Walck
Department of Physics
Lebanon Valley College
Annville, Pennsylvania, USA
`walck@lvc.edu`

To learn quantum mechanics, one must become adept in the use of various mathematical structures that make up the theory; one must also become familiar with some basic laboratory experiments that the theory is designed to explain. The laboratory ideas are naturally expressed in one language, and the theoretical ideas in another. We present a method for learning quantum mechanics that begins with a laboratory language for the description and simulation of simple but essential laboratory experiments, so that students can gain some intuition about the phenomena that a theory of quantum mechanics needs to explain. Then, in parallel with the introduction of the mathematical framework on which quantum mechanics is based, we introduce a calculational language for describing important mathematical objects and operations, allowing students to do calculations in quantum mechanics, including calculations that cannot be done by hand. Finally, we ask students to use the calculational language to implement a simplified version of the laboratory language, bringing together the theoretical and laboratory ideas.

1 Introduction

The theories of twentieth-century physics employ mathematical objects that are quite removed from our everyday experience of the world and surprisingly removed from the description of the experiments that led to or provided evidence for those theories. Certainly theoretical concepts have motivated and guided experiments—experimental design is awash in theory—but if we consider the simplest description of an experiment, as a chef might write a recipe for a lay cook, the language would not include references to the abstract objects that structure the theorist's calculations.

We focus in this paper on the theory of quantum mechanics, and in particular on the behavior of spin-1/2 particles, some of the very simplest quantum systems which nevertheless contain the essential features of quantum mechanics.

The Haskell-based method for learning quantum mechanics takes place within a senior-level course called Physics 421, Quantum Mechanics I, at Lebanon Valley College. Students in the course may have no experience with Haskell or programming at all.

We take the attitude of Papert[4] and others[8, 9, 1, 12] that students are aided in their learning by having building blocks with which to create interesting structures, that such creative activity is a motivating and effective way to learn, and that the feedback provided by computer-language-based building blocks can expose our confusions and produce delight in our achievements.

The remainder of the paper is organized as follows. In section 2, we introduce a laboratory language for the description of experiments with spin-1/2 particles. In section 3, we describe a calculational language for working with kets and operators, the abstract objects used to do calculations. In section 4, we describe a simplified laboratory language that students are asked to implement using the calculational language.

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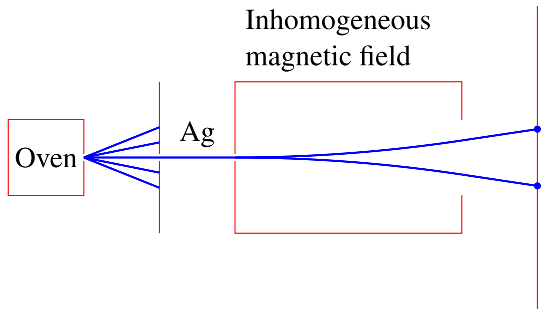


Figure 1: The Stern-Gerlach experiment.

2 Laboratory Language

2.1 Stern-Gerlach Experiment

The Stern-Gerlach experiment was performed by Otto Stern and Walther Gerlach in 1922, a time when the theory of quantum mechanics was being developed. The experiment demonstrates the quantization of angular momentum, which we now know comes in integer and half-integer multiples of $\hbar$, Planck's constant. Several modern quantum mechanics textbooks begin the subject with the Stern-Gerlach experiment.[7, 10, 6] The Feynman Lectures also introduce the Stern-Gerlach experiment early in the volume on quantum mechanics.[2]

A schematic of the Stern-Gerlach experiment is shown in Figure 1. Silver (Ag) atoms are heated in an oven and made into a beam by passing through small holes. Stern and Gerlach chose silver because, with a single electron in its outer shell, it mimics the magnetic behavior of an electron. The electron and the silver atom each possess a *magnetic dipole moment*, that is, they behave like tiny bar magnets in the presence of a magnetic field. Such a magnetic dipole moment will feel a torque, an urge to rotate, in the presence of a magnetic field, and will moreover feel a force if the magnetic field is *inhomogeneous*, changing from one position in space to another.

The Stern-Gerlach experiment aims to produce a force on the silver atoms with an inhomogeneous magnetic field oriented in a particular direction, say the z direction. The silver atoms in the beam are then expected to deflect upward or downward in the z direction, depending on the extent to which their magnetic dipole moments (vectors oriented along the imagined tiny bar magnets) point in the negative or positive z direction. Classical (pre-quantum) physics predicts that the random distribution of magnetic dipole moments coming from the oven should produce a continuous spectrum of deflection of the silver atom beam.

Instead, what is seen in the experiment is that the beam splits into two beams, and produces two spots on a detecting screen. Gerlach called this "directional quantization"[10], and, since magnetic dipole moment is proportional to angular momentum, we now think of this as quantization of angular momentum. The electron and the silver atom are called spin-1/2 particles because the z-component of angular momentum for particles in one of the two beams is $(1/2)\hbar$, and that in other beam is $-(1/2)\hbar$. Spin-1/2 particles are particles that have two outcomes in a Stern-Gerlach experiment (as such, they are examples of quantum bits, or qubits). Although we won't talk about them in this paper, it may be helpful to know that spin-1 particles have three outcomes in a Stern-Gerlach type experiment, spin-3/2 particles have four outcomes, and so on.

Our aim is to use Stern-Gerlach technology to split beams, recombine beams, and act on single beams

```
data BeamStack
randomBeam :: BeamStack
dropBeam   :: BeamStack -> BeamStack
flipBeams  :: BeamStack -> BeamStack
```

Figure 2: The `BeamStack` data type is a collection of beams organized into a stack. The stack consisting of a single beam coming out of an oven is called `randomBeam`. The function `dropBeam` removes the top beam from the stack. The function `flipBeams` interchanges the order of the top two beams on the stack.

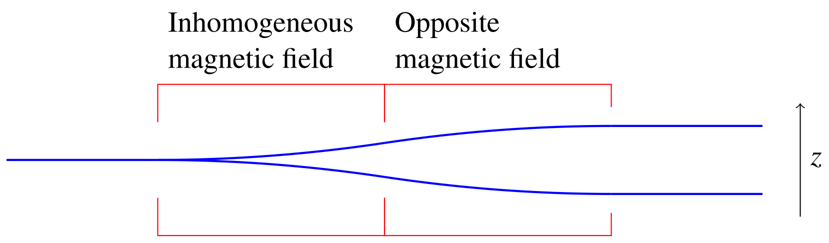
with uniform magnetic fields. For this purpose, we now introduce the central data type in the laboratory language, the `BeamStack`. As in an RPN (reverse Polish notation) calculator, the data type maintains a stack of beams to be acted on in various ways. Figure 2 shows `BeamStack` as an opaque data type. Since the primary pedagogical purpose of the laboratory language is to use it to explore what can happen in experimental setups that the user can design, the focus is not on the implementation of the `BeamStack` data type. Also shown in Figure 2 is an initial `BeamStack`, called `randomBeam`, for the single beam coming out of the oven, and a couple of utility functions to manipulate the stack. These functions and the rest of the laboratory language are available in the module `Physics.Learn.BeamStack` in the *learn-physics* package[11].

We can learn quite a bit more from the Stern-Gerlach experiment if we can do sequential Stern-Gerlach measurements, that is if we can take one of the outcoming beams from the inhomogeneous magnetic field and send it into another Stern-Gerlach device. For this purpose it is helpful to have a Stern-Gerlach splitter that creates two parallel beams. Such a device is shown in Figure 3(a). The key to making the beams parallel is to put an oppositely directed inhomogeneous magnetic field immediately after the first field to deflect the beams back to parallel.

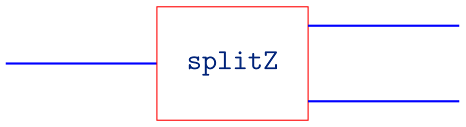
A schematic view of the SG (Stern-Gerlach) beam splitter is shown in Figure 3(b), which is labeled with the laboratory language function `splitZ`. The `splitZ` function takes a `BeamStack` as input, pops the top beam off of the stack, and replaces it with two new beams. The lower beam on the right side of the splitter (the *spin-down* beam) is placed on the top of the stack. Figure 3(c) lists functions for beam splitters in various directions.

The quantum mechanics book by Townsend[10] gives a sequence of SG experiments that help to show what a theory of quantum mechanics needs to explain, or at least predict. Townsend's Experiment 1 is designed to show that although there is randomness in the measurement of spin-1/2 particles, there is not complete randomness in every measurement. In Experiment 1, shown in Figure 4, the z-spin-up beam of the first SG splitter goes into a second SG splitter oriented in the same direction. The results show that when a beam of z-spin-up particles enter a z-splitter, the entire beam comes out with z-spin-up. The intensity of 0.0 in the z-spin-down beam coming out of the second splitter represents a beam of no particles, or no beam at all. Part (b) of Figure 4 shows the use of the laboratory language in GHCi to carry out Experiment 1. The stack is shown so that the top beam on the stack is printed last, or at the bottom of the printed list. We use the `dropBeam` function because we have no further use for the z-spin-down beam exiting the first splitter. It is not necessary to drop the beam; we could have flipped the beams instead to act with the second splitter on the beam we want while continuing to include all beams in the stack.

In Experiment 1, a combination of splitting and dropping occurs that is called *filtering*. The first splitter is used to filter the beam for particles that have spin-up in the z direction. The filtering operation happens often enough that it is useful to name it. Figure 5 shows several filtering functions that we will use in upcoming experiments.



(a)

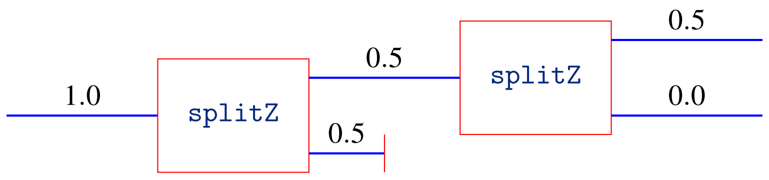


(b)

```
splitX :: BeamStack -> BeamStack
splitY :: BeamStack -> BeamStack
splitZ :: BeamStack -> BeamStack
split  :: Double -> Double -> BeamStack -> BeamStack
```

(c)

Figure 3: The Stern-Gerlach beam splitter. (a) A Stern-Gerlach splitter oriented in the z direction. (b) Schematic representation of the splitter in the z direction, using the `splitZ` function from part (c) of the figure. (c) Laboratory language functions for Stern-Gerlach beam splitters oriented in the x, y, and z directions. The `split` function takes two spherical coordinates as arguments so that the splitting can be done in an arbitrary direction. These functions act on the top (most recent) beam of the stack, remove that beam from the stack, and replace it with two new beams.



(a)

```
GHCi, version 7.10.2: http://www.haskell.org/ghc/  :? for help
Prelude> :m Physics.Learn.BeamStack
Prelude Physics.Learn.BeamStack> randomBeam
Beam of intensity 1.0
Prelude Physics.Learn.BeamStack> splitZ it
Beam of intensity 0.5
Beam of intensity 0.5
Prelude Physics.Learn.BeamStack> dropBeam it
Beam of intensity 0.5
Prelude Physics.Learn.BeamStack> splitZ it
Beam of intensity 0.5
Beam of intensity 0.0
```

(b)

Figure 4: Townsend's[10] Experiment 1. (a) Measuring the same thing twice in succession gives the same results. Every particle that is found to deflect in the positive z direction at the first splitter also deflects in the positive z direction at the second splitter. We see this from the intensities. The entire intensity of 0.5 that enters the second splitter comes out with positive deflection. The intensity of 0.0 in the negatively deflected beam means that no particles are deflected in the negative z direction at the second splitter. (b) A GHCi transcript showing use of the laboratory language to obtain the results of Experiment 1. The beam at the top of the stack is the last beam printed and hence appears at the bottom of the list.

```
xpFilter :: BeamStack -> BeamStack
xpFilter = dropBeam . splitX

xmFilter :: BeamStack -> BeamStack
xmFilter = dropBeam . flipBeams . splitX

zpFilter :: BeamStack -> BeamStack
zpFilter = dropBeam . splitZ

zmFilter :: BeamStack -> BeamStack
zmFilter = dropBeam . flipBeams . splitZ
```

Figure 5: Filtering is the composition of splitting and dropping. The function `xpFilter` keeps only the beam that deflected in the positive x direction. Since the beam that deflected in the negative x direction is placed on the top of the stack in the `split` function, we merely have to drop it. The function `xmFilter` keeps only the beam that deflected in the negative x direction. Since the beam that deflected in the negative x direction is placed on the top of the stack in the `split` function, we need to flip the beams on the stack so that we drop the positive x beam. The functions `ypFilter` and `ymFilter` could, of course, also be defined.

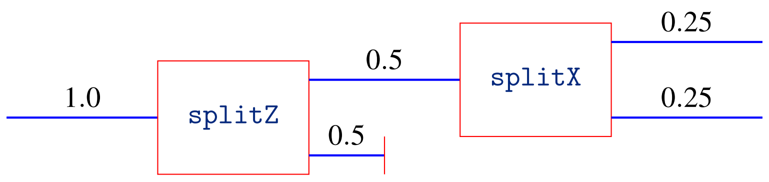
In Townsend's Experiment 2, shown in Figure 6, the second z-splitter of Experiment 1 is replaced by an x-splitter. We see that an incoming beam of z-spin-up particles experiences a 50/50 split at the x-splitter.

Townsend's Experiment 3 extends Experiment 2 with a third splitter, so that the orientations of the splitters are z then x then z. At the output of the last splitter, we now have equal intensities of z-spin-up and z-spin-down beams. This result may be surprising or puzzling when compared with Experiment 1, in which a second z-splitter sends all of the particles in the direction in which they split at a previous z-splitter. In Experiment 3, all of the particles entering the last z-splitter had previously split upward at the first z-splitter, yet now half of those entering the last z-splitter are splitting downward. Experiment 3 can also be viewed as inserting a filter for x-spin-up particles between the splitters of Experiment 1. Clearly this filter is having a significant and strange effect on the final splitting. It seems as though the particles have "forgotten" that they had previously split upwards at a z-splitter. This is a crucial observation that will need to be reflected in the theory.

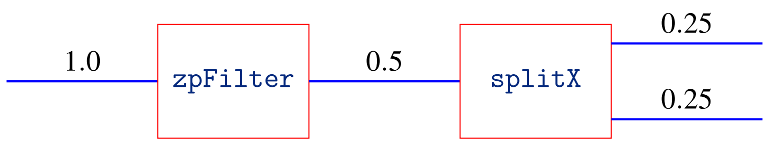
Experiment 3 shows that the upper beam exiting the x-splitter will undergo a 50/50 split at the subsequent z-splitter. It is also the case, as can be tested with the laboratory language, that the lower beam exiting the x-splitter will also undergo a 50/50 split if sent into a z-splitter.

We can recombine two beams with the same kind of inhomogeneous magnetic fields that we used to split a beam. Figure 8 shows an SG recombiner. Recombining is not as intuitive as it might seem. If the two beams that enter a recombiner did not come from a splitter in the same direction, there is no guarantee that they will bend the right way to merge them into a single beam. For example, flipping two beams before recombining will generally give different results (and often a beam intensity of zero) from simply recombining the two beams. This is something that students found quickly in their explorations.

Townsend's Experiment 4, shown in Figure 9, builds on Experiment 3 by adding an x-recombiner after the x-splitter. The surprising result here is that we recover the results of Experiment 1. The final z-splitter produces no z-spin-down particles, just as the final z-splitter in Experiment 1 produced no z-



(a)

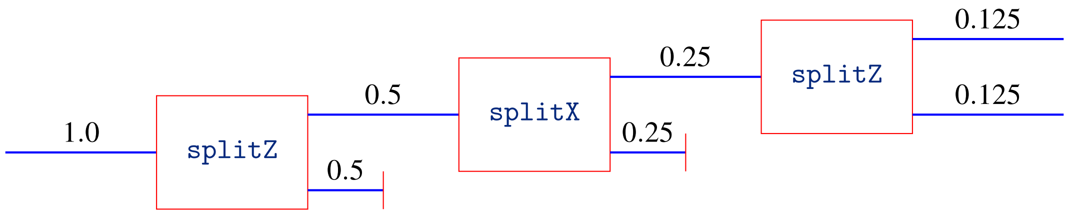


(b)

```
GHCi, version 7.10.2: http://www.haskell.org/ghc/  :? for help
Prelude> :m Physics.Learn.BeamStack
Prelude Physics.Learn.BeamStack> zpFilter randomBeam
Beam of intensity 0.5
Prelude Physics.Learn.BeamStack> splitX it
Beam of intensity 0.25000000000000006
Beam of intensity 0.24999999999999994
```

(c)

Figure 6: Townsend's Experiment 2. (a) Schematic diagram with splitters. (b) Alternate diagram of the same experiment using a filter. (c) GHCi transcript.

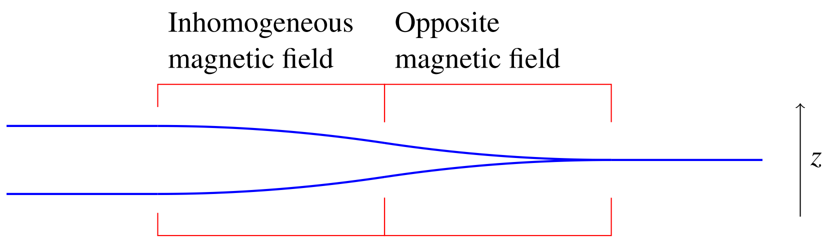


(a)

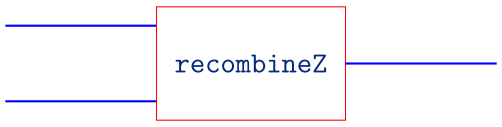
```
Prelude Physics.Learn.BeamStack> randomBeam
Beam of intensity 1.0
Prelude Physics.Learn.BeamStack> splitZ it
Beam of intensity 0.5
Beam of intensity 0.5
Prelude Physics.Learn.BeamStack> dropBeam it
Beam of intensity 0.5
Prelude Physics.Learn.BeamStack> splitX it
Beam of intensity 0.25000000000000006
Beam of intensity 0.24999999999999994
Prelude Physics.Learn.BeamStack> dropBeam it
Beam of intensity 0.25000000000000006
Prelude Physics.Learn.BeamStack> splitZ it
Beam of intensity 0.12500000000000006
Beam of intensity 0.125
```

(b)

Figure 7: Townsend's Experiment 3. Some particles split downward at the last z-splitter, even though all particles entering the last z-splitter have previously split upward at the first z-splitter. This tempers the results of Experiment 1, which showed that no particles would split downward at the second z-splitter after they had split upward at the first z-splitter. Reconciling Experiments 1 and 3 is an important job for the theory. (a) Schematic diagram with splitters. (b) GHCi transcript. There are many alternate ways to do this, including using filters.



(a)

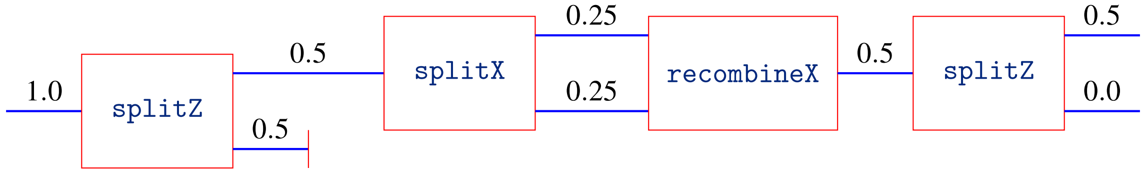


(b)

```
recombineX :: BeamStack -> BeamStack
recombineY :: BeamStack -> BeamStack
recombineZ :: BeamStack -> BeamStack
recombine  :: Double -> Double -> BeamStack -> BeamStack
```

(c)

Figure 8: The Stern-Gerlach beam recombiner. (a) A Stern-Gerlach recombiner oriented in the z direction. (b) Schematic representation of the recombiner in the z direction, using the `recombineZ` function from part (c) of the figure. (c) Stern-Gerlach beam recombiners oriented in the x, y, and z directions. The `recombine` function takes two spherical coordinates as arguments so that the recombining can be done in an arbitrary direction. These functions act on the top two beams of the stack, remove those beams from the stack, and replace them with a single new beam.



(a)

```
Prelude Physics.Learn.BeamStack> randomBeam
Beam of intensity 1.0
Prelude Physics.Learn.BeamStack> splitZ it
Beam of intensity 0.5
Beam of intensity 0.5
Prelude Physics.Learn.BeamStack> dropBeam it
Beam of intensity 0.5
Prelude Physics.Learn.BeamStack> splitX it
Beam of intensity 0.25000000000000006
Beam of intensity 0.24999999999999994
Prelude Physics.Learn.BeamStack> recombineX it
Beam of intensity 0.5
Prelude Physics.Learn.BeamStack> splitZ it
Beam of intensity 0.5
Beam of intensity 0.0
```

(b)

Figure 9: Townsend's Experiment 4. One way to view this experiment is that an x-splitter-recombiner pair has been inserted into the apparatus of Experiment 1, and the same results are obtained as in Experiment 1. A second way to view this experiment is that two beams, each of which would produce a 50/50 split in a `splitZ`, are being recombined into a beam that produces a 100/0 split in a `splitZ`.

spin-down particles. Whereas the x-splitter in Experiment 3 disturbed the repeatability of the z-splitter results in Experiment 1, the x-splitter-recombiner combination in Experiment 4 does not disturb the repeatability. This is another issue that the theory will need to deal with.

Of course we can use the laboratory language to go beyond Townsend's experiments. Students can invent other configurations of splitters and recombiners and see if the results match their expectations.

The last basic building block of the laboratory language is application of a uniform magnetic field to a beam. A magnetic field can be applied in a particular direction with a certain strength for a certain amount of time. The functions to do this are as follows.

```
applyBFieldX :: Double -> BeamStack -> BeamStack
applyBFieldY :: Double -> BeamStack -> BeamStack
applyBFieldZ :: Double -> BeamStack -> BeamStack
applyBField  :: Double -> Double -> Double -> BeamStack -> BeamStack
```

These functions apply a uniform magnetic field to the top beam of the stack. In the function `applyBFieldX`, the `Double` argument is an angle in radians that represents a combination of the strength of the applied magnetic field and the duration over which it is applied. The `applyBField` function takes two spherical coordinates as arguments to represent the direction of the applied magnetic field, and a third numerical argument to represent the combination of magnetic field strength and time over which the field is applied.

Here are some example puzzles for students to work on as they explore the laboratory language.

- First, find a sequence of two filters such that no particles exit the second filter (no particles is the same as a beam of zero intensity). Now, is it possible to find a third filter to place between the first two, such that particles now flow from the last filter? If this is possible, we may need to adjust our intuition about what a filter is.
- Can you find a direction and duration for a uniform magnetic field to act on a beam exiting a `zpFilter` so that the entire beam intensity will make it through an `xpFilter`? Does this suggest a way to think about what a uniform magnetic field does?
- In Townsend's Experiment 4, suppose we apply a uniform magnetic field in the x direction to the lower beam between the x-splitter and x-recombiner. If the duration of application of the magnetic field is zero, the results will match that of Experiment 4. What is the next shortest duration when the results match again? Is the answer surprising?

Students seemed happy to play with the laboratory language, and some came up with interesting situations. I gave almost no instruction on how to use GHCi or Haskell, yet students seemed to make good progress. That function application takes precedence over operations such as division was not intuitive for my students. Most surprising to me was that while students asked questions about a variety of issues, no one sought clarification about why they were getting errors from the compiler. They just tried different things until something worked. I learned that I need to be proactive about explaining error messages.

3 Calculational Language

3.1 Postulates of Quantum Mechanics

A summary of the rules of quantum mechanics can be given in the form of a set of postulates.

Vector Space Postulate: Associated with each quantum system there is a complex inner product space V.

State Postulate: The *state* of the quantum system is described by a vector in V. The zero vector does not describe any state. Two vectors that are multiples of one another describe the same state.

Observable Postulate: An *observable* is described by a Hermitian linear operator on V.

Evolution Postulate: In the absence of intervention, a state vector $|\psi(t)\rangle$ evolves according to the Schrödinger equation,

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = \hat{H}\,|\psi(t)\rangle\,,$$

where $\hat{H}$ is a special observable called the *Hamiltonian*.

Composition Postulate: Suppose that quantum system 1 is associated with complex inner product space V_1 and that quantum system 2 is associated with complex inner product space V_2. The quantum system composed of both system 1 and system 2 is associated with complex inner product space $V_1 \otimes V_2$, the tensor product of spaces V_1 and V_2.

`data Ket`	
`xp :: Ket`	$\|x_+\rangle = \frac{1}{\sqrt{2}}\|z_+\rangle + \frac{1}{\sqrt{2}}\|z_-\rangle$
`xm :: Ket`	$\|x_-\rangle = \frac{1}{\sqrt{2}}\|z_+\rangle - \frac{1}{\sqrt{2}}\|z_-\rangle$
`yp :: Ket`	$\|y_+\rangle = \frac{1}{\sqrt{2}}\|z_+\rangle + \frac{i}{\sqrt{2}}\|z_-\rangle$
`ym :: Ket`	$\|y_-\rangle = \frac{1}{\sqrt{2}}\|z_+\rangle - \frac{i}{\sqrt{2}}\|z_-\rangle$
`zp :: Ket`	$\|z_+\rangle$
`zm :: Ket`	$\|z_-\rangle$
`np :: Double -> Double -> Ket`	$\|n_+(\theta,\phi)\rangle = \cos\frac{\theta}{2}\|z_+\rangle + e^{i\phi}\sin\frac{\theta}{2}\|z_-\rangle$
`nm :: Double -> Double -> Ket`	$\|n_-(\theta,\phi)\rangle = \sin\frac{\theta}{2}\|z_+\rangle - e^{i\phi}\cos\frac{\theta}{2}\|z_-\rangle$

Figure 10: Kets for spin-1/2 particles.

Measurement Postulate:

1. The possible outcomes of a measurement of an observable $\hat{A}$ are the *eigenvalues* of $\hat{A}$.
2. The probability of obtaining outcome a in a measurement of $\hat{A}$ is given by $\langle\psi|\hat{P}_a|\psi\rangle$, where $|\psi\rangle$ is the normalized state of the system prior to measurement, and $\hat{P}_a$ is the projection operator associated with outcome a.

Projection Postulate: Measuring a system changes its state. If $|\psi\rangle$ is the state of a system prior to a measurement then $\hat{P}_a|\psi\rangle$ is the state of the system after the measurement, where a is the measured value of the observable.

The Vector Space Postulate says that for each system we might want to study and describe with quantum mechanics, we need a complex vector space of a certain dimension, which could be finite or infinite. We call this vector space a complex inner product space because we need to be able to take the inner product of vectors.

The State Postulate claims that vectors in this complex inner product space describe states of the quantum system. In the case of a spin-1/2 particle, a vector can describe the state of the particle as it exits a Stern-Gerlach splitter, for example. Paul Dirac created a notation based on the inner product, or bracket $\langle\phi|\psi\rangle$, calling $\langle\phi|$ a bra and $|\psi\rangle$ a ket. The bra vector is dual to the ket vector. Jerzy Karczmarczuk gave an early implementation of kets in Haskell.[3] Figure 10 shows ket vectors for spin-1/2 particles. The calculational language is available in the module `Physics.Learn.Ket` in the *learn-physics* package[11].

The Observable Postulate says that physical quantities such as position, momentum, angular momentum, and energy are represented by linear operators in quantum mechanics. For a spin-1/2 particle, the observables of interest are components of angular momentum in various directions. We denote by S_x, S_y, and S_z the x-, y-, and z-components of spin angular momentum. The Pauli operators σ_x, σ_y, and σ_z are then defined by $S_x = \frac{\hbar}{2}\sigma_x$, $S_y = \frac{\hbar}{2}\sigma_y$, and $S_z = \frac{\hbar}{2}\sigma_z$. These Pauli operators are implemented as `sx`, `sy`, and `sz` in Figure 11.

In addition to kets, bras, and operators, the calculational language has a generic multiplication and an adjoint operation. These are shown in Figure 12.

The Evolution Postulate governs how the state ket of a system changes over time. The state ket satisfies the *Schrödinger equation*, in which a Hamiltonian operator $\hat{H}$ must be specified to describe the system and its interaction with the environment. The calculational language provides a function

```
timeEv :: Double -> Operator -> Ket -> Ket
```

`data Operator`	
`sx :: Operator`	$\lvert x_+\rangle\langle x_+\rvert - \lvert x_-\rangle\langle x_-\rvert$
`sy :: Operator`	$\lvert y_+\rangle\langle y_+\rvert - \lvert y_-\rangle\langle y_-\rvert$
`sz :: Operator`	$\lvert z_+\rangle\langle z_+\rvert - \lvert z_-\rangle\langle z_-\rvert$
`sn :: Double -> Double -> Operator`	$\lvert n_+(\theta,\phi)\rangle\langle n_+(\theta,\phi)\rvert - \lvert n_-(\theta,\phi)\rangle\langle n_-(\theta,\phi)\rvert$

Figure 11: Operators for spin-1/2 particles.

Term	Type	Notation
`xp`	`Ket`	$\lvert x_+\rangle$
`dagger ym`	`Bra`	$\langle y_-\rvert$
`dagger ym <> xp`	`Complex Double`	$\langle y_-\lvert x_+\rangle$
`yp <> dagger ym`	`Operator`	$\lvert y_+\rangle\langle y_-\rvert$
`sx <> yp`	`Ket`	$\sigma_x\lvert y_+\rangle$
`dagger zm <> sy <> yp`	`Complex Double`	$\langle z_-\rvert\sigma_y\lvert y_+\rangle$

Figure 12: Examples of constructions with kets, bras, and operators. The third line is an inner product. The fourth line is an outer product. The function `dagger` is an adjoint operation that turns kets into bras, bras into kets, operators into (adjoint) operators, and complex numbers into their complex conjugates. The generic multiplication <> is used for scalar products, inner products, outer products, operators products, and wherever it makes sense.

to numerically solve the Schrödinger equation, using an algorithm from *Numerical Recipes*[5]. This function takes a time step, a Hamiltonian operator, and the current state ket, and returns the state ket advanced by the time step.

The Measurement Postulate gives rules for obtaining the possible outcomes of a measurement and the probability of each possible outcome. While projection operators are an elegant way to state this rule in general, there are many situations in which a measurement result is associated with an *outcome ket* $\lvert\phi\rangle$. In these simplified situations, the probability of the outcome is given by the square of the magnitude of the inner product of the outcome ket with the state ket $\lvert\psi\rangle$.

$$P = \lvert\langle\phi\lvert\psi\rangle\rvert^2$$

In the calculational language, we would express this as follows.

```
magnitude (dagger phi <> psi) ** 2
```

Here, `phi` and `psi` are kets, `dagger phi` is a bra vector, and <> is the generic multiplication used for scalar multiplication, inner product (the case here), and outer product.

4 Simplified Laboratory Language

Having used the calculational language to solve problems, make predictions, and do animations, we would now like to use it to implement the laboratory language that we started with. However, the laboratory language that we started with requires one important feature that we have not included in the calculational language, a feature that I do not typically cover in a one-semester course on quantum mechanics. What is missing is the idea of a density matrix, which is a more general way of describing the

```
data Beam
xpBeam :: Beam
xmBeam :: Beam
ypBeam :: Beam
ymBeam :: Beam
zpBeam :: Beam
zmBeam :: Beam
split  :: Double -> Double -> Beam -> (Beam,Beam)
splitX :: Beam -> (Beam,Beam)
splitY :: Beam -> (Beam,Beam)
splitZ :: Beam -> (Beam,Beam)
xpFilter :: Beam -> Beam
xmFilter :: Beam -> Beam
zpFilter :: Beam -> Beam
zmFilter :: Beam -> Beam
recombine  :: Double -> Double -> (Beam,Beam) -> Beam
recombineX :: (Beam,Beam) -> Beam
recombineY :: (Beam,Beam) -> Beam
recombineZ :: (Beam,Beam) -> Beam
applyBField  :: Double -> Double -> Double -> Beam -> Beam
applyBFieldX :: Double -> Beam -> Beam
applyBFieldY :: Double -> Beam -> Beam
applyBFieldZ :: Double -> Beam -> Beam
intensity :: Beam -> Double
```

Figure 13: Simplified version of the laboratory language. The goal is for a student to implement the `Beam` data type and the listed functions in terms of the calculational language.

state of a physical system. The only purpose to which the density matrix is put in the laboratory language is the description of the `randomBeam` coming out of the oven.

Our simplified laboratory language will constrain itself to beams that have already come out of some SG apparatus. There is a great simplification in this constraint, in that our central data type can now be `Beam`, rather than `BeamStack`. The functions in Figure 13 are those that students are asked to implement. In terms of the `Beam` data type, their type signatures are clearer and more meaningful than the corresponding functions for working with a `BeamStack`.

Up to this point, students have not really been programming. They have been using GHCi as a fancy calculator. Very little needed to be explained in term of language syntax and semantics. At this point, some time needs to be spent on basic language issues in order for students to be able to write function definitions for the functions in Figure 13. Exactly how much time to spend and what to introduce when is still a work in progress.

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